Modeling 1D Cold Electrostatic Plasma with a Lagrangian Particle Method

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Abstract

The conventional approach to plasma simulations utilizes the particle-in-cell (PIC) method, but PIC simulations often lose resolution as complex features appear in the evolving plasma. This project develops an alternative Lagrangian particle method for a one-dimensional cold electrostatic plasma with periodic boundary conditions, which is both efficient and preserves accuracy as the plasma evolves in time, in contrast to existing methods. The plasma is described by the Vlasov-Poisson equations for the electron distribution in phase space and the self-consistent electric field in physical space. The plasma is represented by discrete charged macro-particles (representing electrons) with a neutralizing constant background distribution of positively charged ions. Two integration techniques, Euler's method and fourth order Runge-Kutta, are used to evolve the electrons, and regularization is applied to ensure continuity of the electric field. We investigate the effect of the numerical parameters, including the time step Δt, number of particles N, and the regularization parameter δ. Initial results indicate that the lack of continuity in the non-regularized problem contributes to error growth for both integration schemes. Additionally, a particle insertion scheme is implemented to preserve the resolution of the plasma. While this method fails to converge properly for the case of repulsive particles, it shows promise for the case of attractive particles, with possible applications in the study of dark matter at cosmological scales. Future extensions of this work seek to apply the method to study other distributions, like the two-stream instability and warm distributions, and further investigate the effects of electric field regularization regarding energy conservation and charge transport.

Introduction and Methodology

- Plasma is present in several applications across various fields, including space exploration, medicine, energy production, propulsion, etc.
- This project makes the following simplifying assumptions:
  - One dimensional plasma
  - Cold plasma: no velocity distribution, only spatial.
  - Spatial periodic boundary conditions on interval [0,1].
  - Electrostatic plasma: slow particle motion without magnetic forces.
- A plasma distribution $f(x,v,t)$ subject to the electrostatic force $F = qE$, electric field $E = -\partial_x \varphi$, and electric potential φ is modeled using the Vlasov-Poisson equations:

$$\partial_t f + v\partial_x f + F\partial_v f = 0 \qquad E = -\partial_x^2 \varphi$$

- Assuming a stationary, positively charged ion background, the electric field $E(x,t)$ is given in terms of a kernel function $K(x,y)$ by:

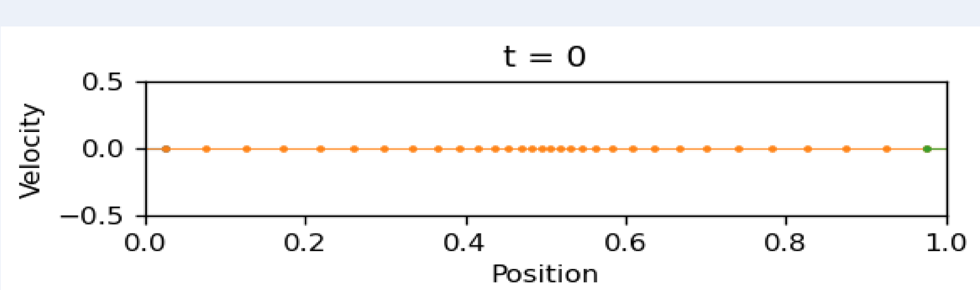
$$E(x,t) = \int_{-\infty}^{\infty}\int_0^1 K(x,y)F(y,v,t)\,dy\,dv \qquad K(x,y) = -\frac{1}{2}\operatorname{sign}(x-y) + x - y$$

- **Lagrangian approach:** Once discretized, the N plasma particles are assigned identifying Lagrangian coordinates α_i, and respective position and velocity functions $x_i(t)$ and $v_i(t)$ for $1 \le i \le N$. These functions are calculated by solving the equations of motion with a discretized electric field

$$x_i''(t) = qE(x_i(t),t) = -\sum_{j=1}^{N} K\left(x_i(t), x_j(t)\right) w_j$$

- These equations are solved using Euler's Method and 4th order Runge-Kutta integration, enforcing periodic boundary conditions.
- Particles start at rest and are given an initial spatial perturbation with magnitude ϵ from an equidistant arrangement:

$$x_i(0) = \alpha_i + \epsilon\sin(2\pi\alpha_i) \quad v_i(t) = 0 \qquad \alpha_i = \frac{i - {}^1\!/_2}{N}$$



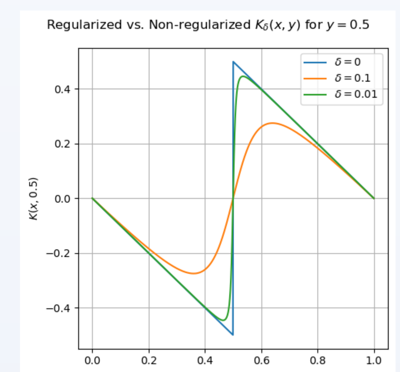
Initial particle arrangement for $N = 30$ particles and $\epsilon = 0.1$

Electric Field Regularization and Adaptive Particle Insertion

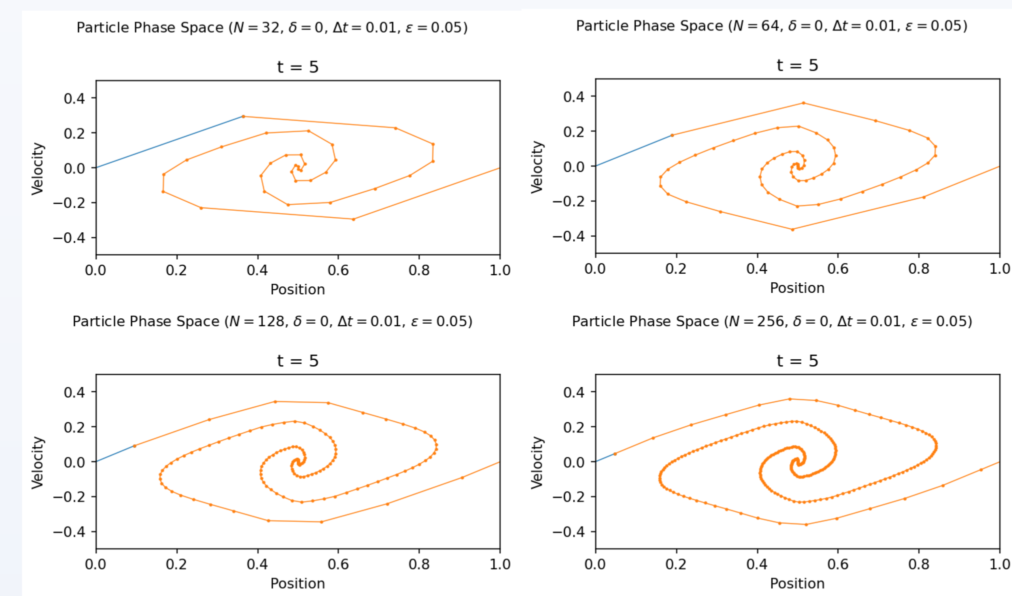
- Implement an adaptive particle insertion scheme to address separation of particles.
- Introduce passive and active particles:
  - Passive particles have no charge and are advected by the flow.
  - Passive particles have a charge and contribute to the plasma's electric field.
- Particles are inserted when distance between passive particles exceeds threshold d_1.
- New particles are inserted using quadratic interpolation of position and velocity with respect to Lagrangian coordinate.

- Introduce regularization parameter δ and regularized kernel $K_\delta(x,y)$ to smooth E:

$$K_\delta(x,y) = \frac{c_\delta}{2}\frac{x-y}{\sqrt{(x-y)^2+\delta^2}} + x - y$$



Electric field kernel K_δ with different regularization parameters δ.



Convergence of plasma distribution at $t = 5$ for increasing number of particles N with no regularization. Notice irregularities are present at the center of the spiral.

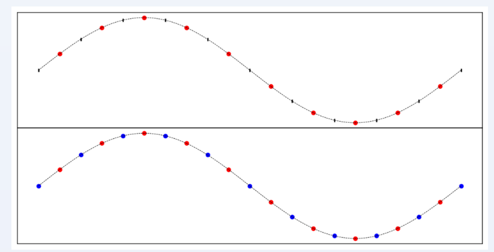


Illustration of particle only discretization (top) vs. active and passive particle discretization (bottom).

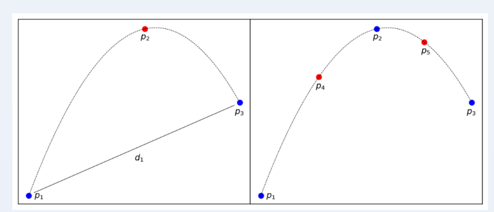
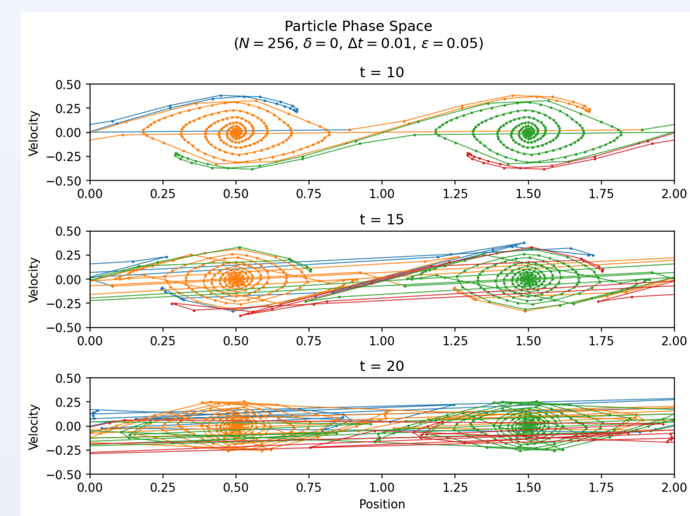
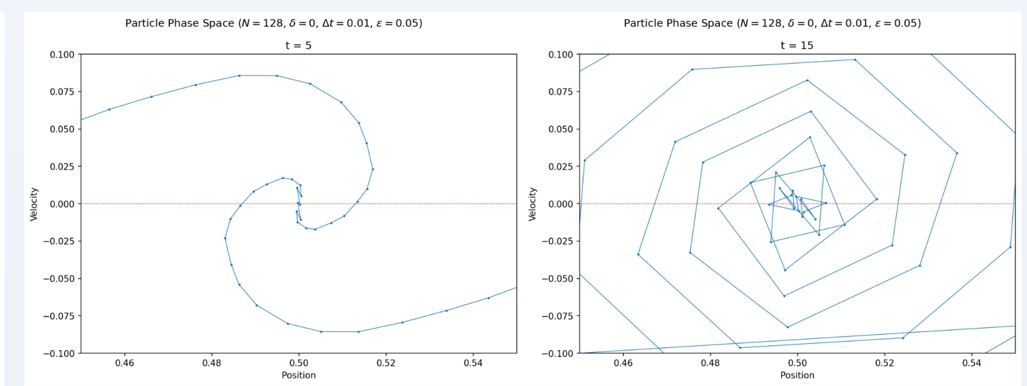


Illustration of adaptive particle insertion before (left) and after (right). Passive particles shown in blue and active particles shown in red.



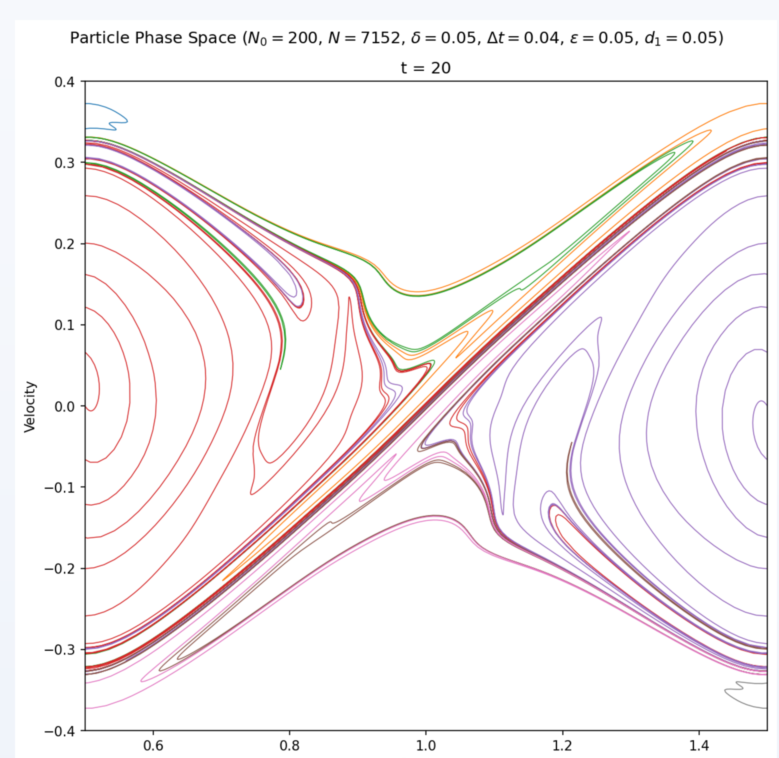
Evolution of plasma after $t = 10$ with lines connecting neighboring particles. No adaptive particle insertion.



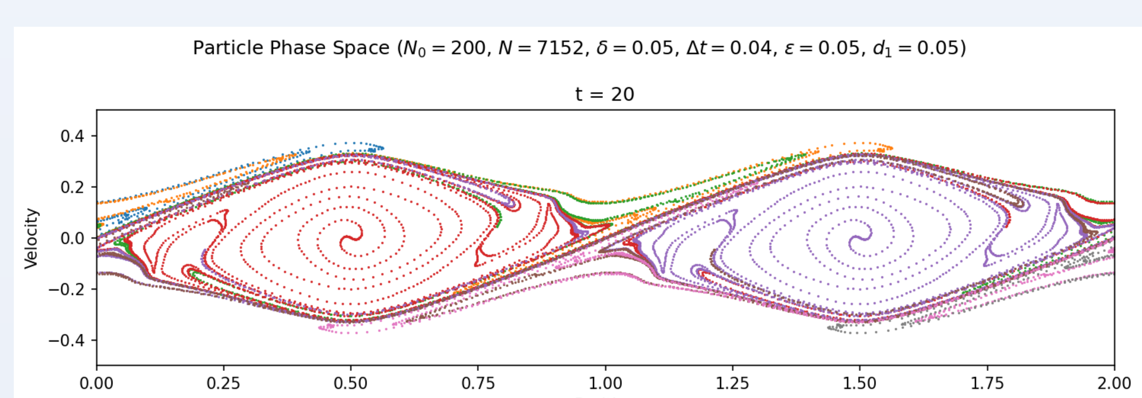
Enlarged view of the center of the plasma distribution without regularization for $N = 128$ at $t = 5$ (left) and $t = 15$ (right).

Results with Regularization and Adaptive Particle Insertion

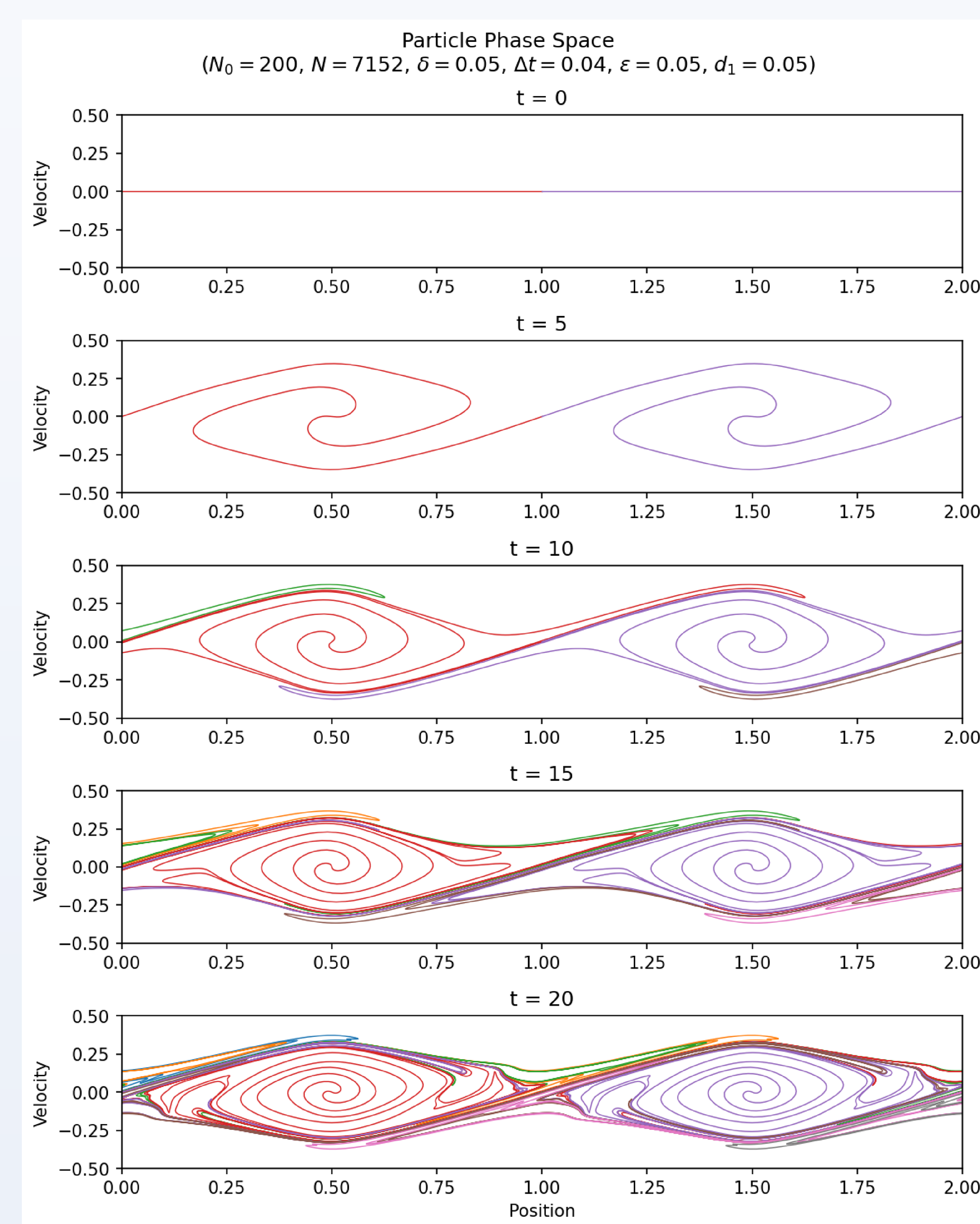
- Regularization and adaptive particle insertion help define small detail.
- Particles are dense only where needed. This helps reduce computational costs.
- Distribution structure is preserved even when connecting lines are removed.
- Particles attract?



Enlarged plot of region between two neighboring periods. Different colors denote particles starting in different periods.

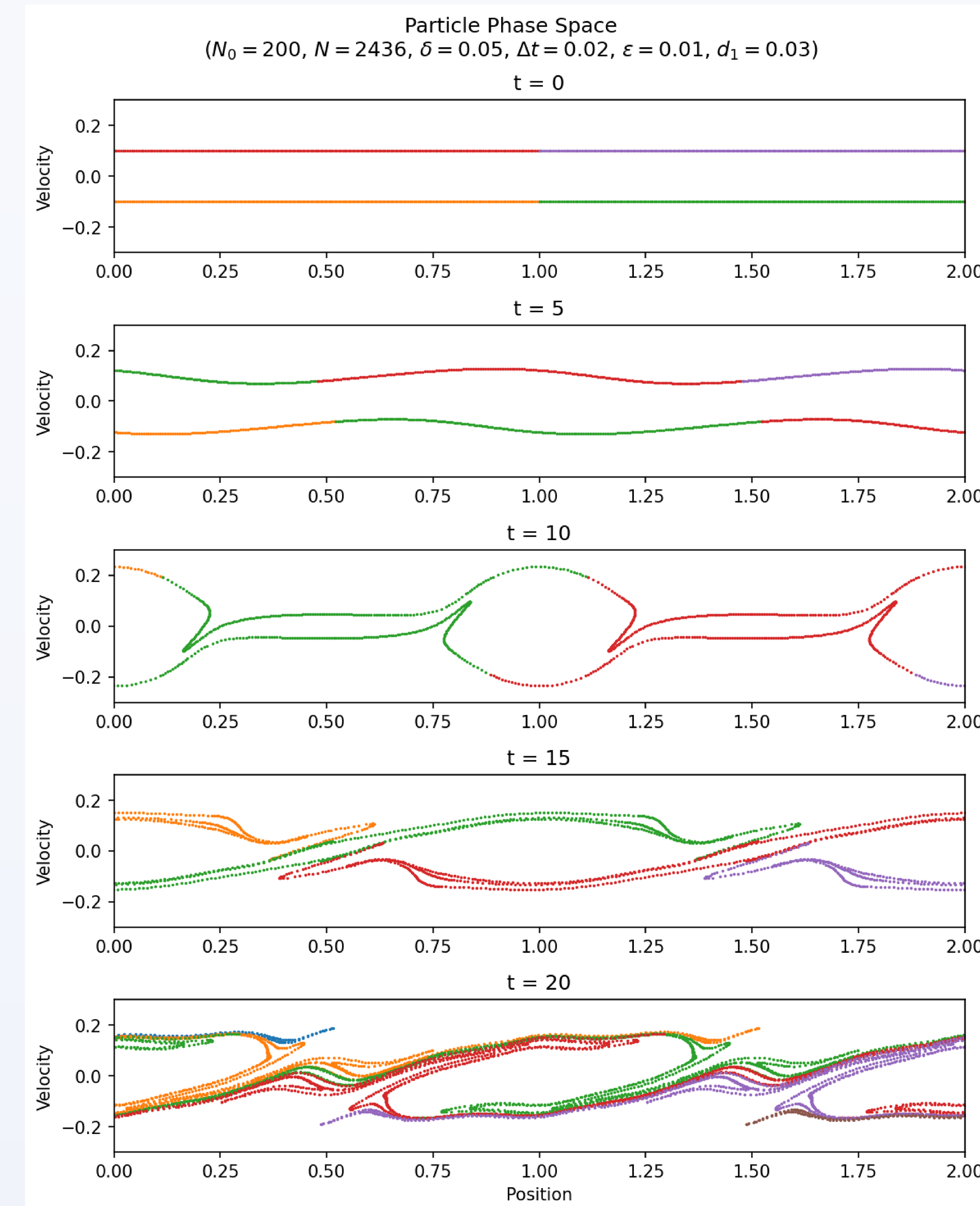


Plasma distribution with $N_0 = 200$ initial particles and $N = 7152$ final particles at $t = 20$ with regularization and adaptive particle insertion with no connecting lines.



Plasma distribution in phase space with $N_0 = 200$ initial particles and $N = 7152$ final particles at times $t = 5$, $t = 10$, $t = 15$, $t = 20$, with regularization and adaptive particle insertion.

Repulsive Force Two-Stream Instability



Plasma distribution in phase space with $N_0 = 200$ initial particles and $N = 2436$ final particles for a two-stream instability with initial velocities $v(0) = \pm 0.1$ for a repulsive electrostatic force. Hard-to-resolve peaks develop in the plasma.

Conclusion and Further Directions

- Explore possible applications in cosmology with an attractive force.
- Compare accuracy of this method vs. other methods for simulating the same problem that use a velocity distribution.
- Investigate analogous methods where plasma is treated as a system of discrete particles rather than as an approximation to a continuous distribution.
  - How to make sense of particle insertion in this context?
  - Investigate two-stream instability and compare with current implementation.

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